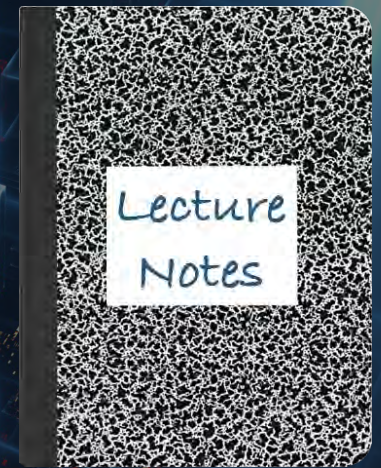


CS 417 – DISTRIBUTED SYSTEMS

**Week 3: Part 3**  
Logical Clocks

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# Logical clocks

## Assign sequence numbers to messages

- All cooperating processes can agree on order of events
- vs. *physical clocks*: report time of day

## Assume no central time source

- Each system maintains its own local clock
- No total ordering of events
  - No concept of *happened-when*
- **Assume multiple actors (processes)**
  - Each process has a unique ID
  - Each process has its own incrementing counter

# Happened-before

## Lamport's “happened-before” notation

$a \rightarrow b$  event  $a$  happened before event  $b$

e.g.:  $a$ : message being sent,  $b$ : message received

Transitive:

if  $a \rightarrow b$  and  $b \rightarrow c$  then  $a \rightarrow c$

# Logical clocks & concurrency

Assign a “clock” value to each event

- if  $a \rightarrow b$  then  $\text{clock}(a) < \text{clock}(b)$  since time cannot run backwards

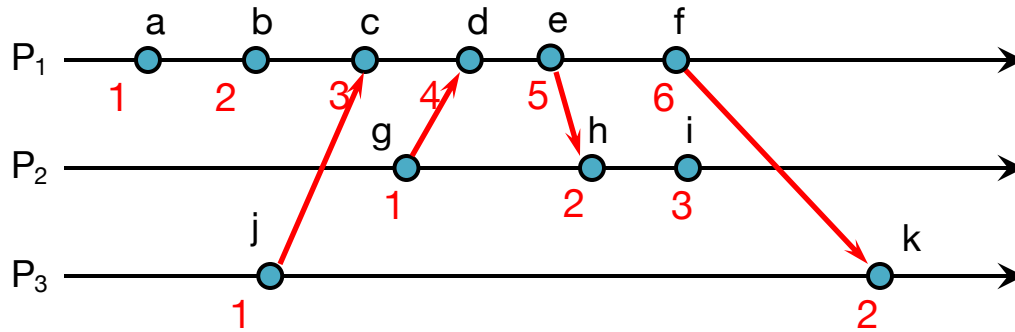
If  $a$  and  $b$  occur on different processes that do not exchange messages, then neither  $a \rightarrow b$  nor  $b \rightarrow a$  are true

- These events are **concurrent**
- Otherwise, they are **causal**

# Event counting example

- Three systems:  $P_1$ ,  $P_2$ ,  $P_3$
- Events  $a$ ,  $b$ ,  $c$ , ...
- Local event counter on each system
- Systems occasionally communicate

# Event counting example



Bad ordering:

$e \rightarrow h$  but  $5 \geq 2$

$f \rightarrow k$  but  $6 \geq 2$

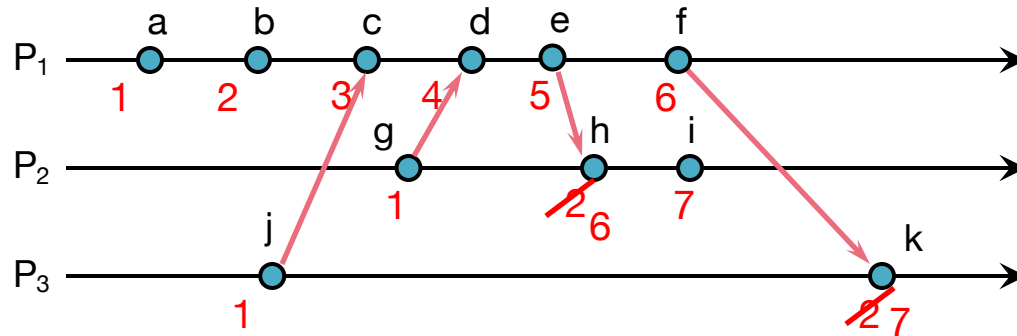
# Lamport Timestamps

- Each process has its own clock (sequence #)
- Clock is incremented before each event
- Each message carries a timestamp of the sender's clock
- When a message arrives:
  - if receiver's *clock*  $\leq$  *message\_timestamp*
  - set system clock to (*message\_timestamp* + 1)
  - set event timestamp to the system's clock

Lamport timestamps allow us to maintain time ordering among related events  $\Rightarrow$  **Partial ordering**

# Event counting example

*Applying Lamport timestamps*



We have good ordering where we used to have bad ordering:

$e \rightarrow h$  and  $5 < 6$

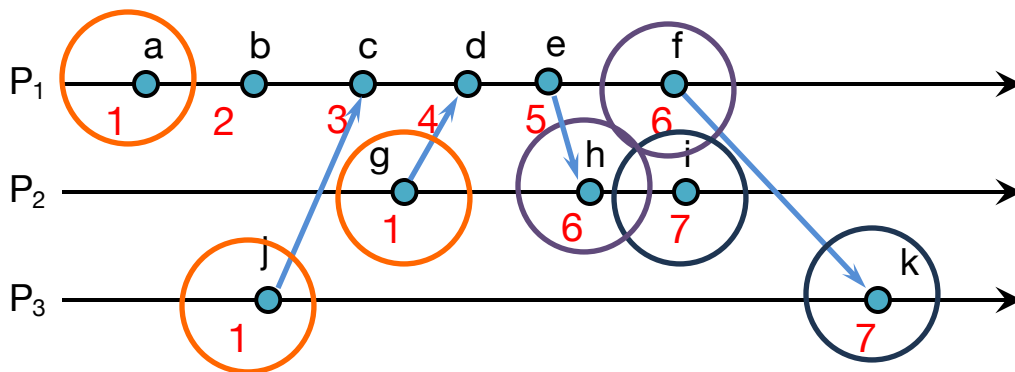
$f \rightarrow k$  and  $6 < 7$



# Summary

- Lamport timestamps need a monotonically increasing software counter
- Incremented when events that need to be timestamped occur
  - Every message that is sent contains the timestamp
  - Every received message sets the clock to  $\max(\text{msg\_timestamp} + 1, \text{clock})$
  - The event is associated with the value of the clock (**Lamport timestamp**)
- For any two events, where  $a \rightarrow b$ :  
$$L(a) < L(b)$$

# Problem: Identical timestamps



$a \rightarrow b, b \rightarrow c, \dots$  :

local events sequenced

$i \rightarrow c, f \rightarrow d, d \rightarrow g, \dots$  :

Lamport imposes a  
*send*  $\rightarrow$  *receive* relationship

**Concurrent events (e.g.,  $b$  &  $g$ ;  $i$  &  $k$ ) may have the same timestamp ... *or not***

# Unique timestamps (total ordering)

## We can force each timestamp to be unique

- Define global logical timestamp  $(T_i, i)$ 
  - $T_i$  represents local Lamport timestamp
  - $i$  represents a globally unique process number
    - e.g., (host address, process ID)
- Compare timestamps:

$$(T_i, i) < (T_j, j)$$

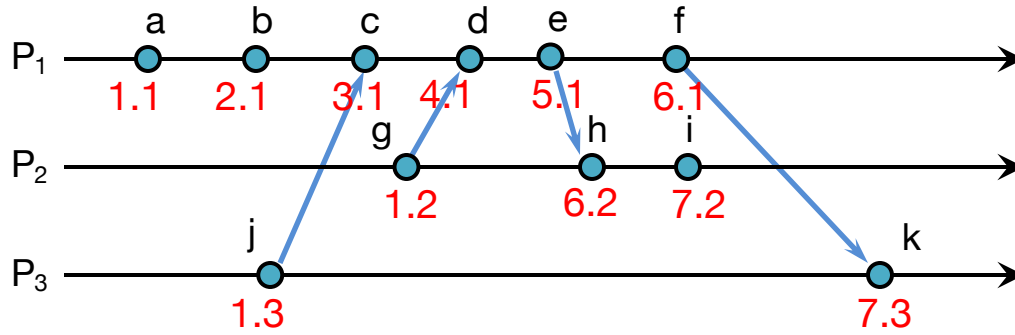
if and only if

$$T_i < T_j \text{ or}$$

$$T_i = T_j \text{ and } i < j$$

**Does not necessarily relate to actual sequence of events**

# Unique (totally ordered) timestamps



# Problem: Detecting causal relations

If  $L(e) < L(e')$

- We cannot conclude that  $e \rightarrow e'$

By looking at Lamport timestamps

- We cannot conclude which events are causally related

Solution: use a **vector clock**

Vector clocks are a way to prove the sequence of events by keeping a version history based on each process that created an event

# Example

- Group of processes: *Alice, Bob, Cindy, David*
- They send messages to decide: “*what food should we eat?*”
- Each process keeps a local counter

Alice writes the value & sends to group



Bob reads ("Pizza", <alice:1>), modifies the value & sends to group



*Bob's version updates Alice's choice*

Receivers

**<alice: 1, bob:1>** is causal to & follows **<alice: 1>**

Alice reads ("Chinese", <alice:1, bob:1>), modifies the value & sends to group

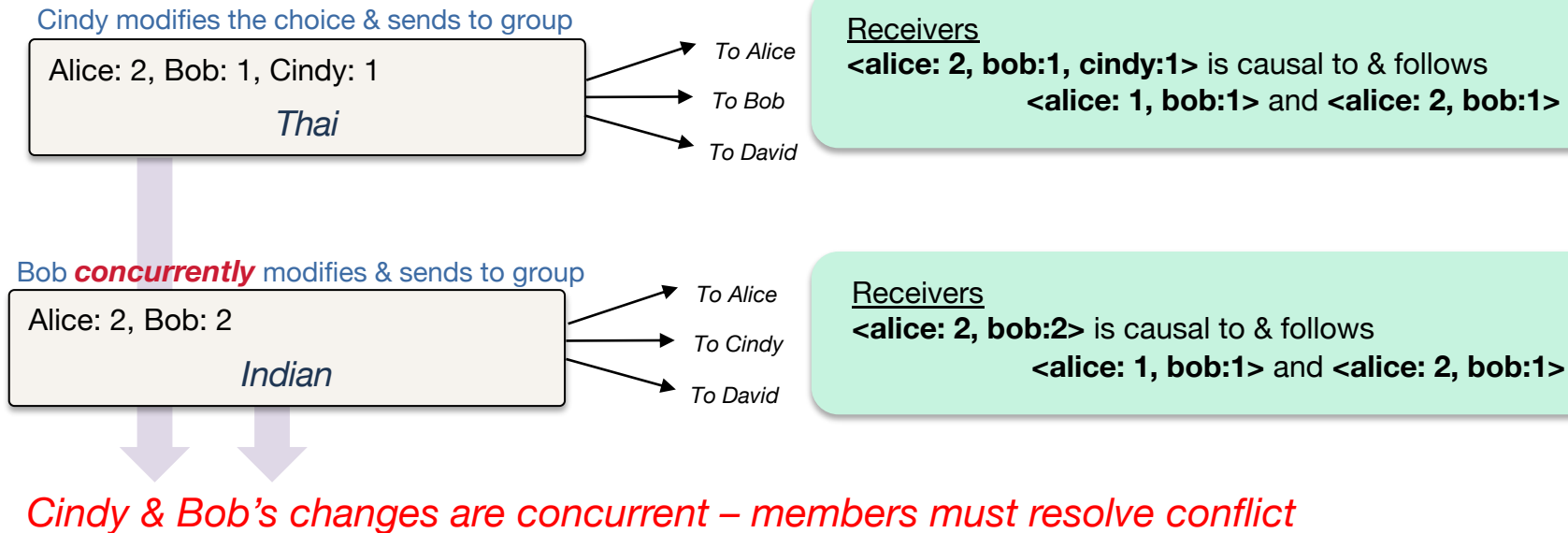


*Alice makes changes over Bob's choice*

Receivers

**<alice: 2, bob:1>** is causal to & follows **<alice: 1, bob:1>**

# Example



Receiver  
**<alice: 2, bob:1, cindy:1>** is concurrent with **<alice: 2, bob:2>**

# Vector clocks: Rules

1. Vector initialized to 0 at each process  $i$  for  $N$  processes

$$V_i[j] = 0 \text{ for } i, j = 1, \dots, N$$

2. Process increments its element of the vector in local vector before timestamping event:

$$V_i[i] = V_i[i] + 1$$

3. Message is sent from process  $P_i$  with  $V_i$  attached to it

4. When  $P_j$  receives message, compares vectors element by element and sets local vector to higher of two values

$$V_j[i] = \max(V_i[i], V_j[i]) \text{ for } i = 1, \dots, N$$

For example,

We received:  $[0, 5, 12, 1]$ , we currently have:  $[2, 8, 10, 1]$

The time vector will be updated to:  $[2, 8, 12, 1]$



# Comparing vector timestamps

## Define

$V = V'$  iff  $V[i] = V'[i]$  for  $i = 1 \dots N$

$V < V'$  iff  $V \neq V'$  and  $V[i] \leq V'[i]$  for  $i = 1 \dots N$

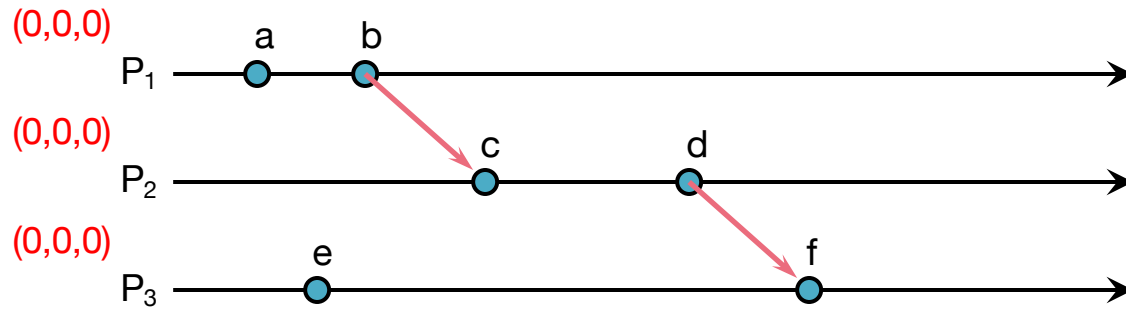
For any two events  $e, e'$

if  $e \rightarrow e'$  then  $V(e) < V(e')$  ... just like Lamport timestamps

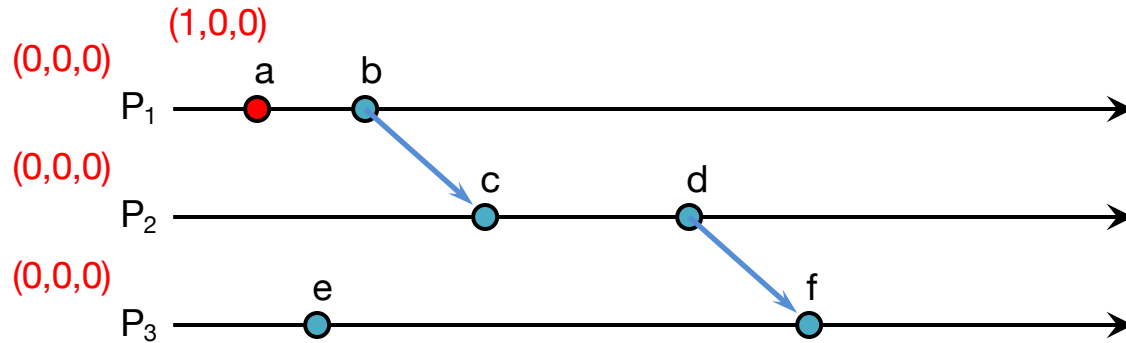
if  $V(e) < V(e')$  then  $e \rightarrow e'$

Two events are **concurrent** if **neither**  $V(e) < V(e')$  **nor**  $V(e') < V(e)$

# Vector timestamps

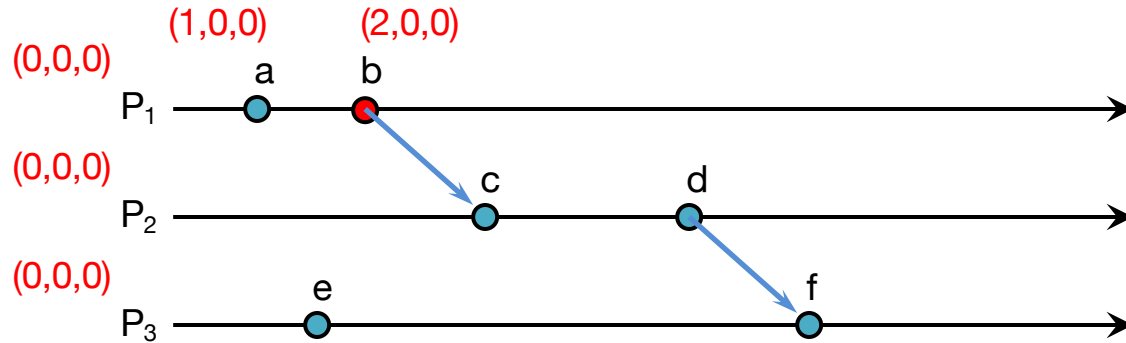


# Vector timestamps



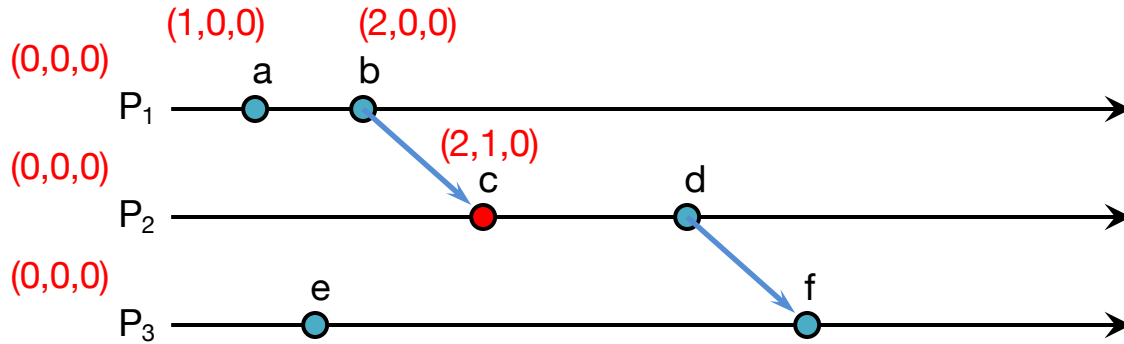
<u>Event</u>	<u>timestamp</u>
a	$(1,0,0)$

# Vector timestamps



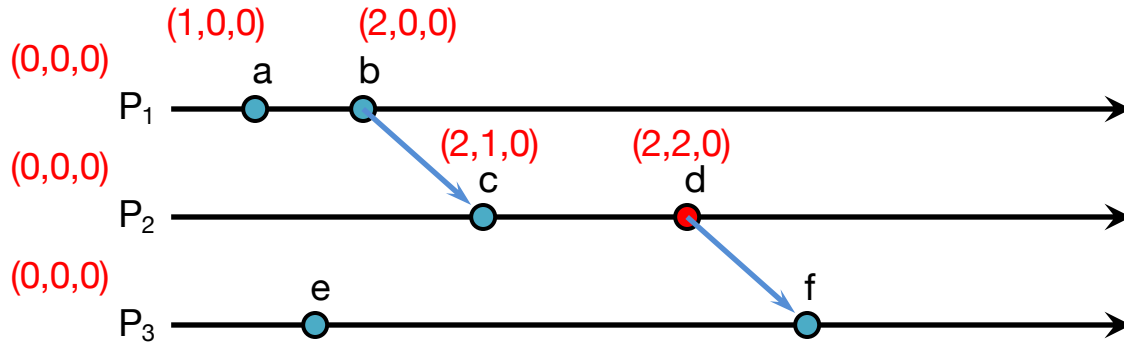
<u>Event</u>	<u>timestamp</u>
a	$(1,0,0)$
b	$(2,0,0)$

# Vector timestamps



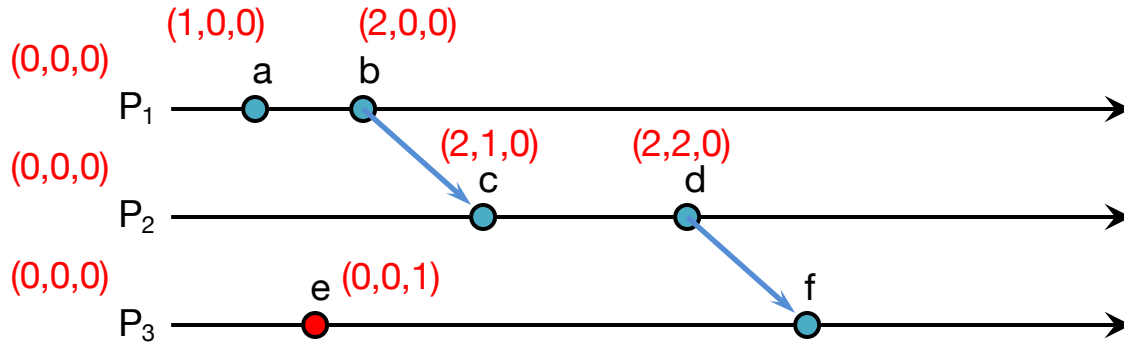
<u>Event</u>	<u>timestamp</u>
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)

# Vector timestamps



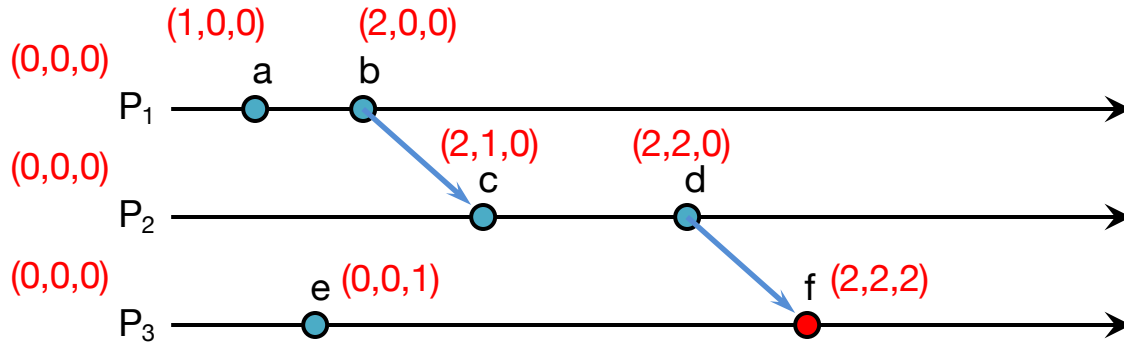
<u>Event</u>	<u>timestamp</u>
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)

# Vector timestamps



<u>Event</u>	<u>timestamp</u>
a	$(1,0,0)$
b	$(2,0,0)$
c	$(2,1,0)$
d	$(2,2,0)$
e	$(0,0,1)$

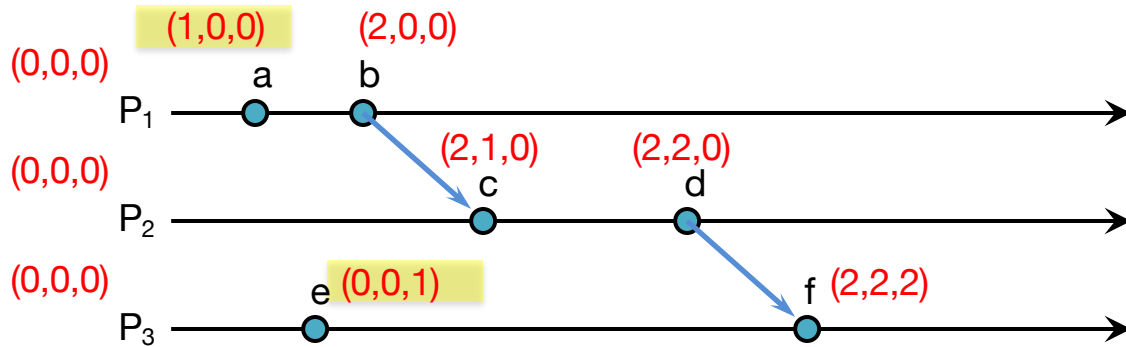
# Vector timestamps



<u>Event</u>	<u>timestamp</u>
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)
e	(0,0,1)
f	(2,2,2)



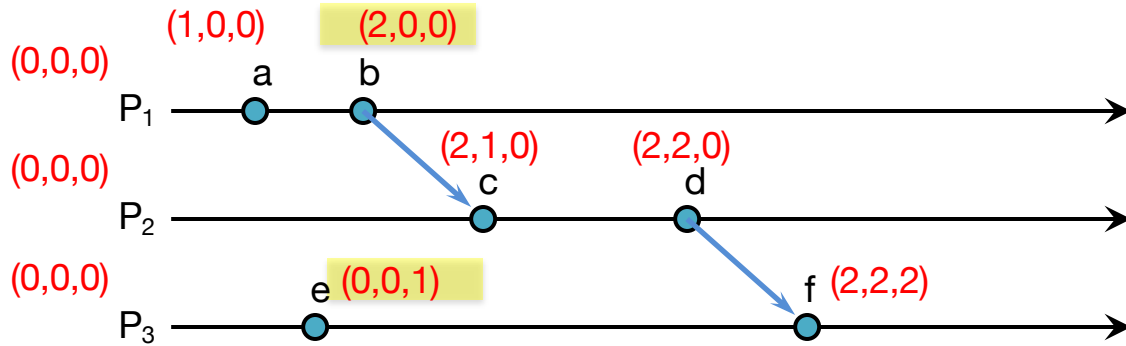
# Vector timestamps



Event	timestamp
a	$(1,0,0)$
b	$(2,0,0)$
c	$(2,1,0)$
d	$(2,2,0)$
e	$(0,0,1)$
f	$(2,2,2)$

*concurrent events*

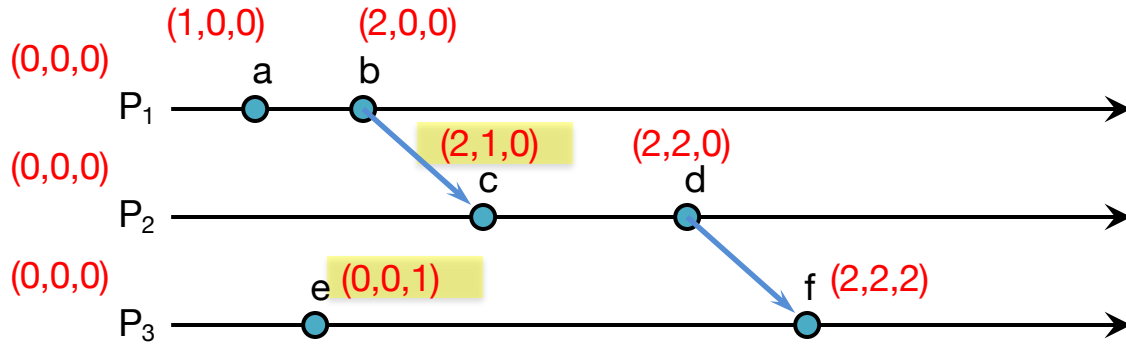
# Vector timestamps



Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)
e	(0,0,1)
f	(2,2,2)

*concurrent events*

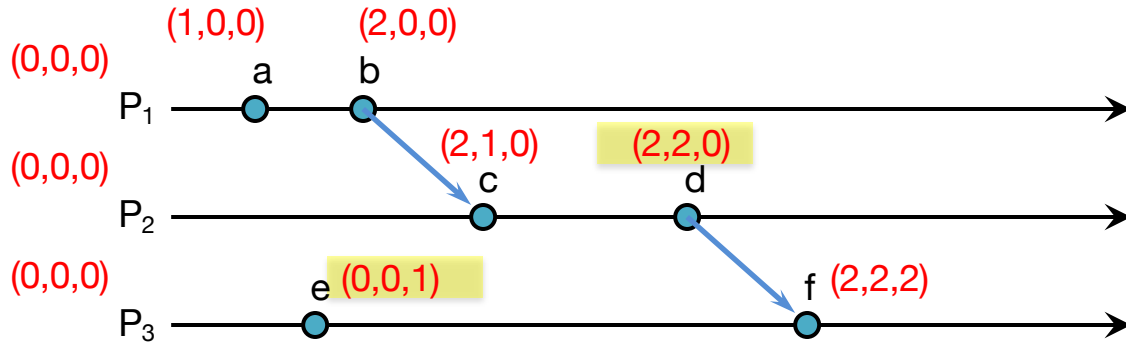
# Vector timestamps



Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)
e	(0,0,1)
f	(2,2,2)

concurrent events

# Vector timestamps



Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)
e	(0,0,1)
f	(2,2,2)

← concurrent events

# Generalizing Vector Timestamps

- **A “vector” can be a list of tuples instead of a vector of numbers:**
  - For processes  $P_1, P_2, P_3, \dots$
  - Each process has a globally unique Process ID,  $P_i$  (e.g.,  $MAC\_address:PID$ )
  - Each process maintains its own timestamp:  $T_{P_1}, T_{P_2}, \dots$
  - Vector:  $\{ \langle P_1, T_{P_1} \rangle, \langle P_2, T_{P_2} \rangle, \langle P_3, T_{P_3} \rangle, \dots \}$
- **One process may only have only partial knowledge of others**
  - New timestamp for a received message:
    - Compare all matching sets of process IDs: set to highest of values
    - Any non-matched  $\langle P, T \rangle$  sets get added to the timestamp
  - For a *happened-before* relation:
    - At least one set of process IDs must be common to both timestamps
    - Match all corresponding  $\langle P, T \rangle$  sets: A:  $\langle P_i, T_a \rangle$ , B:  $\langle P_i, T_b \rangle$
    - If  $T_a \leq T_b$  for all common processes  $P$ , then  $A \rightarrow B$

# Vector Clocks Summary

- Vector clocks give us a way of identifying which events are causally related
- We are guaranteed to get the sequencing correct

But

- The size of the vector increases with more actors  
... and the entire vector must be stored with the data
- Comparison takes more time than comparing two numbers
- What if messages are concurrent?
  - App will have to decide how to handle conflicts

# Summary: Logical Clocks & Partial Ordering

- Causality
  - If  $a \rightarrow b$  then event  $a$  can affect event  $b$
- Concurrency
  - If neither  $a \rightarrow b$  nor  $b \rightarrow a$  then one event cannot affect the other
- Partial Ordering
  - Causal events are sequenced
- Total Ordering
  - All events are sequenced

The End