

# Computer Security

## 08. Cryptography – Part II

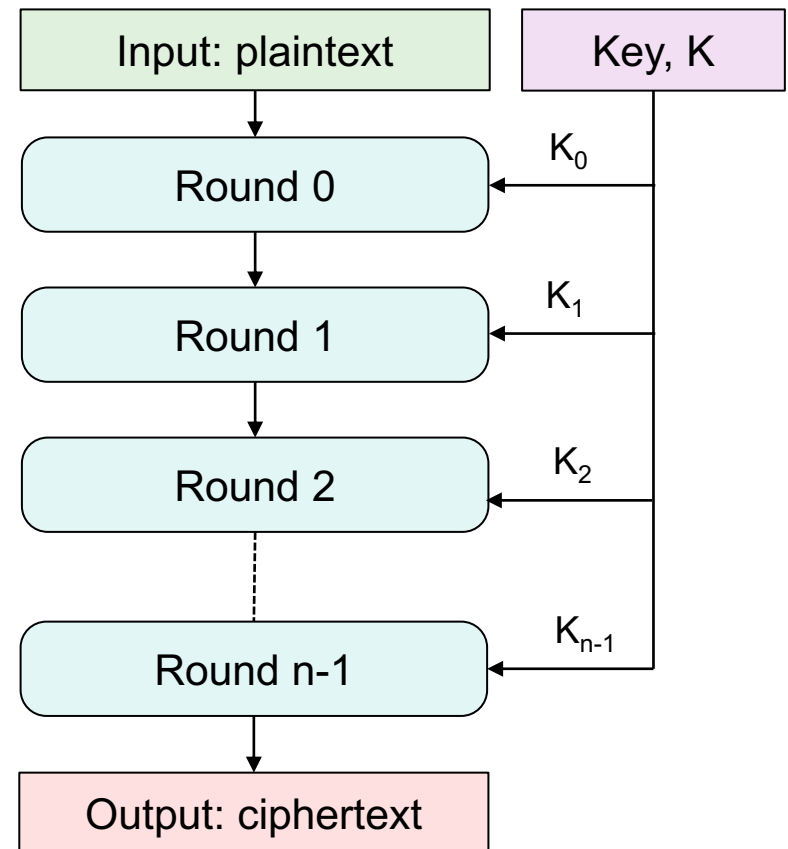
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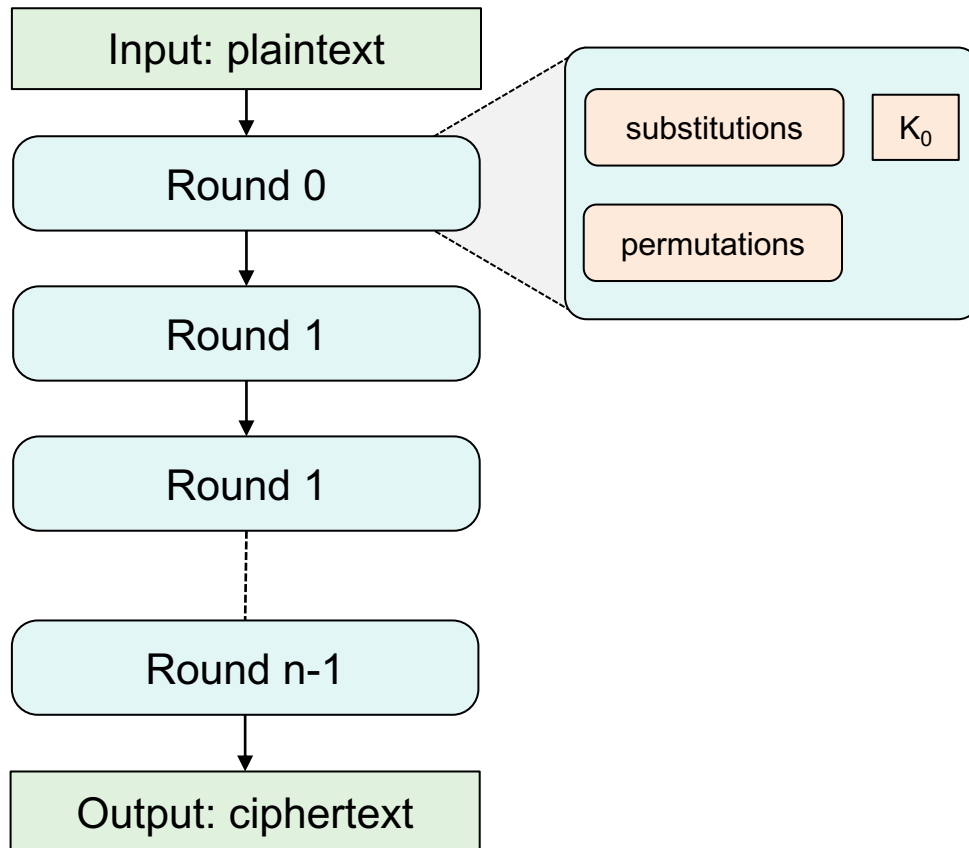
# Block ciphers

- Block ciphers encrypt a *block* of plaintext at a time and produce ciphertext
- **DES** & **AES** are two popular block ciphers
  - DES: 64 bit blocks
  - AES: 128 bit blocks
- Block ciphers are usually **iterative ciphers**
  - The encryption process is an iteration through several *round* operations



# Block cipher rounds

Each round consists of substitutions & permutations



## Substitution = S-box

- Table lookup
- Converts a small block of input to a block of output
- Changing one bit of input should change approximately  $\frac{1}{2}$  of output bits

## Permutation

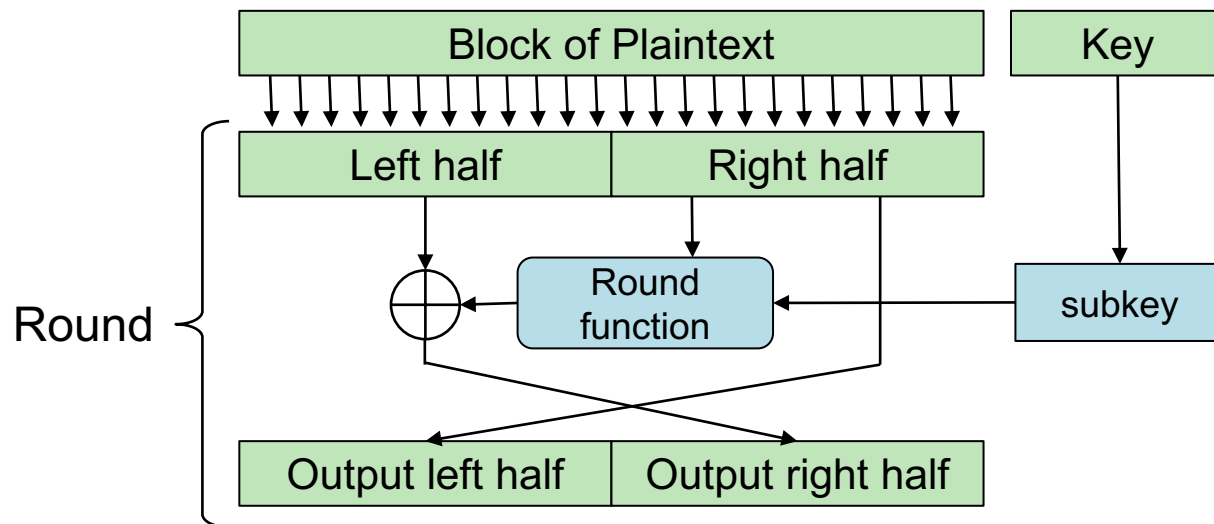
- Scrambles the bits in a prescribed order

## Key application per round

- **Subkey** per round derived from the key
- Can drive behavior of s-boxes
- May be XORed with the output of each round

# Feistel cipher

- DES is a type of **Feistel cipher**, which is a form of a **block cipher**
- Plaintext block is split in two
  - Round function applied to one half of the block
  - Output of the round function is XORed with other half of the block
  - Halves are swapped
- AES is not a Feistel cipher



# AES (Advanced Encryption Standard)

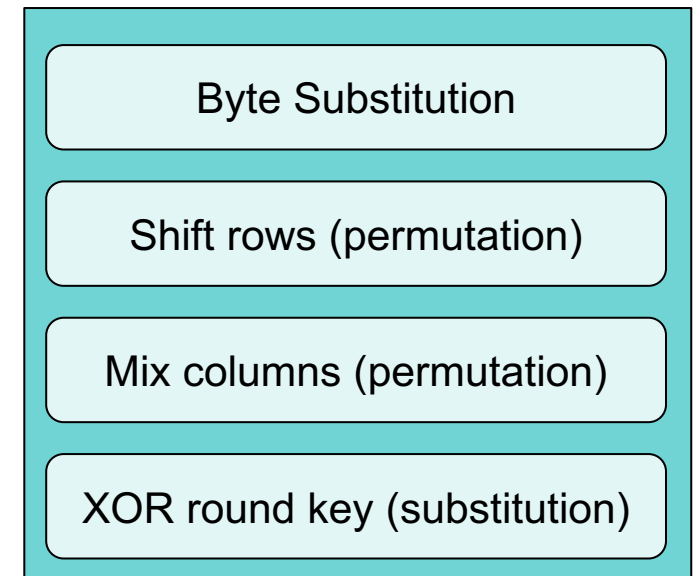
- Block cipher: 128-bit blocks
  - DES used 64-bit blocks
- Successor to DES as a standard encryption algorithm
  - DES: 56-bit key
  - AES: 128, 192, or 256 bit keys

# AES (Advanced Encryption Standard)

- Iterative cipher, just like most other block ciphers
  - Each round is a set of substitutions & permutations
- Variable number of rounds
  - DES always used 16 rounds
  - AES:
    - 10 rounds: 128-bit key
    - 12 rounds: 192-bit key
    - 14 rounds: 256-bit key
  - A **subkey** (“round key”) derived from the key is computed for each round
    - DES did this too

# Each AES Round

- **Step 1: Byte Substitution (s-boxes)**
  - Substitute 16 input bytes by looking each one up in a table (**S-box**)
  - Result is a **4x4 matrix**
- **Step 2: Shift rows**
  - Each row is shifted to the left (wrapping around to the right)
  - 1<sup>st</sup> row not shifted; 2<sup>nd</sup> row shifted 1 position to the left;
  - 3<sup>rd</sup> row shifted 2 positions; 4<sup>th</sup> row shifted three positions
- **Step 3: Mix columns**
  - 4 bytes in each column are transformed
  - This creates a new 4x4 matrix
- **Step 4: XOR round key**
  - XOR the 128 bits of the round key with the 16 bytes of the matrix in step 3



# AES Decryption

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Same rounds  
... but in reverse order



# DES Disadvantages

- DES has been shown to have some weaknesses
  - Key can be recovered using  $2^{47}$  chosen plaintexts or  $2^{43}$  known plaintexts
  - *Note that this is not a practical amount of data to get for a real attack*
- Short block size (8 bytes =  $2^8 = 64$  bits)
- **The real weakness of DES is its 56-bit key**
  - Exhaustive search requires  $2^{55}$  iterations on average
- 3DES solves the key size problem: we can have keys up to 168 bits.
  - Differential & linear cryptanalysis is not effective here: the three layers of encryption use 48 rounds instead of 16 making it infeasible to reconstruct s-box activity.
- DES is relatively slow
  - It was designed with hardware encryption in mind: 3DES is 3x slower than DES
  - *Still much faster than RSA public key cryptosystems!*

# AES Advantages

- **Larger block size:** 128 bits vs 64 bits
- **Larger & varying key sizes:** 128, 192, and 256 bits
  - 128 bits is complex enough to prevent brute-force searches
- **No significant academic attacks beyond brute force search**
  - Resistant against linear cryptanalysis thanks to bigger S-boxes
    - S-box = lookup table that adds non-linearity to a set of bits via transposition & flipping
  - DES: 6-bit inputs & 4-bit outputs
  - AES: 8-bit inputs & 8-bit outputs
- **Typically 5-10x faster in software than 3DES**

# Attacks against AES

- Attacks have been found
  - This does **not** mean that AES is insecure!
- Because of the attacks:
  - AES-128 has computational complexity of  $2^{126.1}$  (~126 bits)
  - AES-192 has computational complexity of  $2^{189.7}$  (~189 bits)
  - AES-256 has computational complexity of  $2^{254.9}$  (~254 bits)
- The security of AES can be increased by increasing the number of rounds in the algorithm
- However, AES-128 still has a sufficient safety margin to make exhaustive search attacks impractical

# Cryptographic attacks

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- Chosen plaintext
  - Attacker can create plaintext and see the corresponding ciphertext
- Known plaintext
  - Attacker has access to both plaintext & ciphertext but doesn't get to choose the text
- Ciphertext-only
  - The attacker only sees ciphertext
  - Popular in movies but rarely practical in real life

# Differential Cryptanalysis

*Examine how changes in input affect changes in output*

- Discover where a cipher exhibits non-random behavior
  - These properties can be used to extract the secret key
  - Applied to block ciphers, stream ciphers, and hash functions (functions that flip & move bits vs. mathematical operations)
- Chosen plaintext attack is normally used
  - Attacker must be able to choose the plaintext and see the corresponding cipher text

# Differential Cryptanalysis

- Provide plaintext with known differences
  - See how those differences appear in the ciphertext
- The properties depend on the **key** and the **s-boxes** in the algorithm
- Do this with lots and lots of known *plaintext-ciphertext* sets
- Statistical differences, if found, may allow a key to be recovered faster than with a brute-force search
  - You may deduce that certain keys are not worth trying

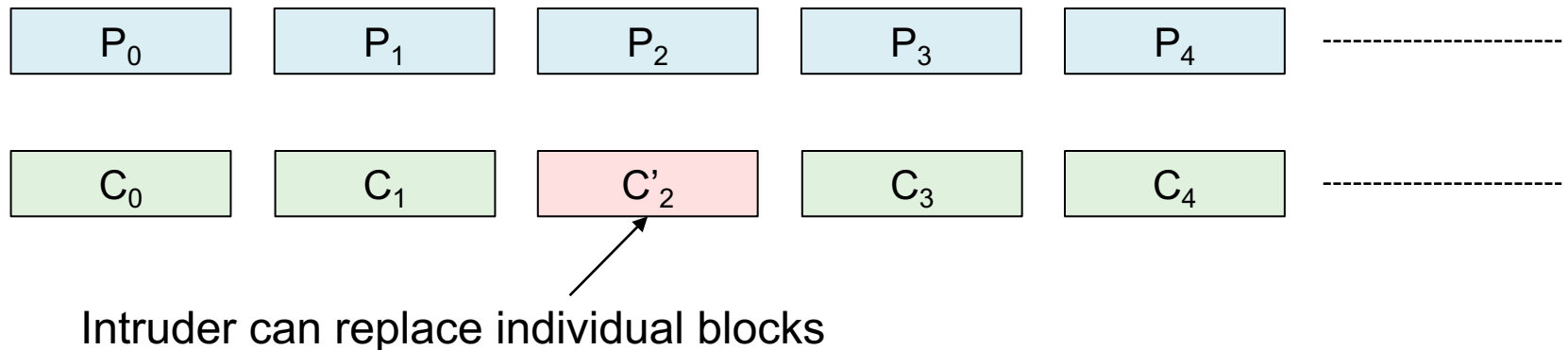
# Linear Cryptanalysis

*Create a predictive approximation of inputs to outputs*

- Instead of looking for differences, linear cryptanalysis attempts to come up with a **linear formula** (e.g., a bunch of xor operations) that **connects certain input bits, output bits, and key bits** with a probability higher than random
  - Goal is to approximate the behavior of s-boxes
- It will not recreate the working of the cipher
  - You just hope to find non-random behavior that gives you insight on what bits of the key might matter
- Works better than differential cryptanalysis for known plaintext  
Differential cryptanalysis works best with chosen plaintext
- Linear & differential cryptanalysis will **rarely recover a key** but may be able to reduce the number of keys that need to be searched.

# Not a good idea to use block ciphers directly

- Streams of data are broken into  $k$ -byte blocks
  - Each block encrypted separately
  - This is called **Electronic Codebook (ECB)**
- Problems
  1. Same plaintext results in identical encrypted blocks  
Enemy can build up a code book of plaintext/ciphertext matches
  2. Attacker can add/delete/replace blocks

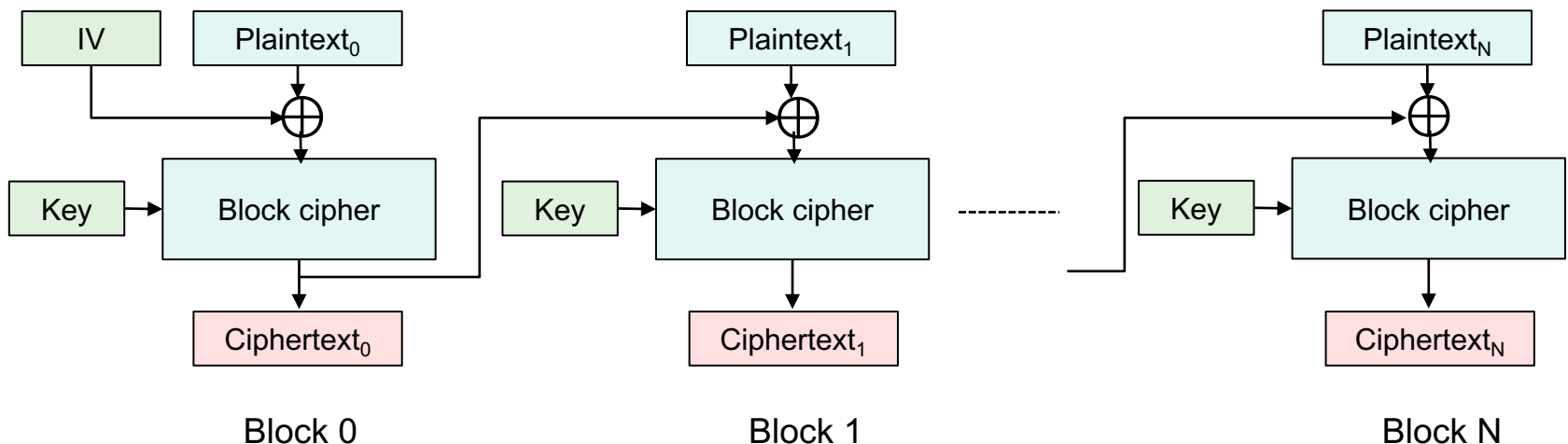




# Cipher Block Chaining (CBC) mode

- Random **initialization vector (IV)** = bunch of  $k$  random bits
  - Non-secret: both parties need to know this
- Exclusive-or with first plaintext block – then encrypt the block
- Take exclusive-or of the result with the next plaintext block

$$c_i = E_K(m) \oplus c_{i-1}$$



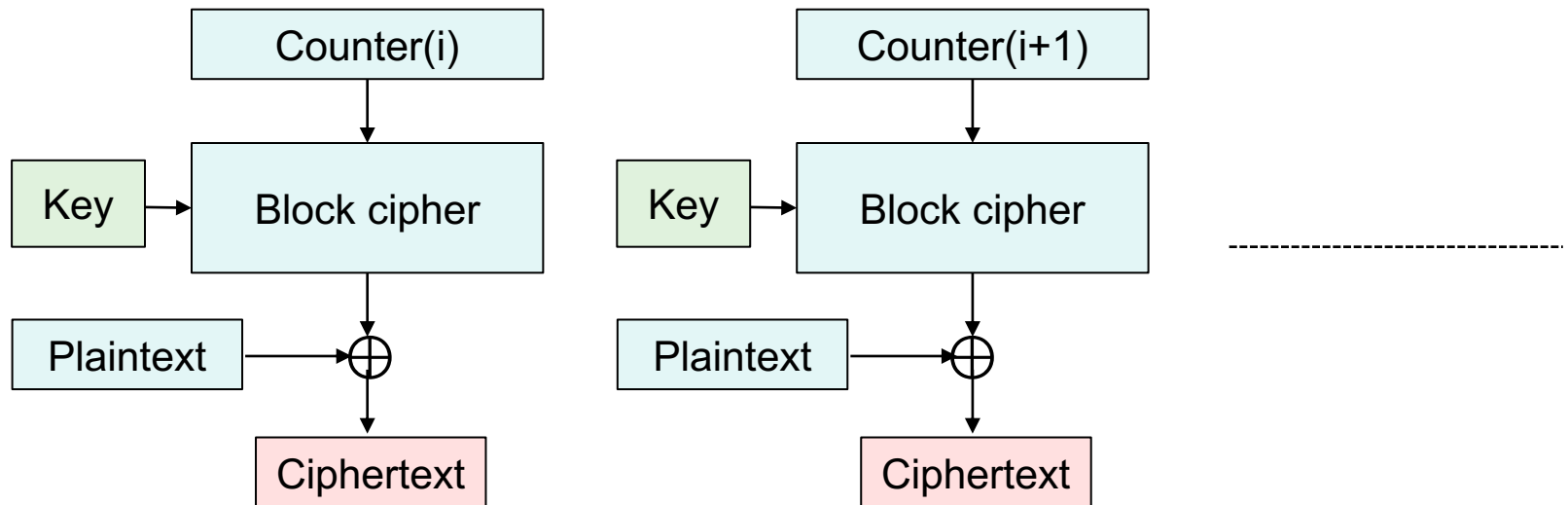
# CBC Observations

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- Identical blocks of plaintext do not produce the same ciphertext
- Each block is a function of all previous blocks
- But an attacker can still cause data corruption

# Block encryption: Counter (CTR) mode

- Random starting **counter** = bunch of  $k$  random bits, just like IV
  - Any function producing a non-repeating sequence (an incrementing number is a common function)
- **Encrypt the counter with the key**
- Exclusive-or result with plaintext block

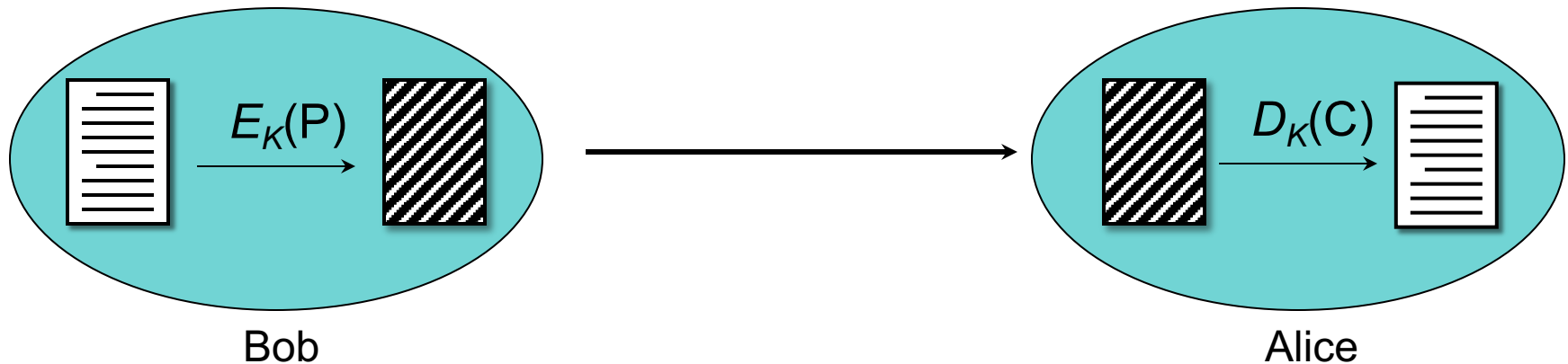


# Popular symmetric algorithms

- AES (Advanced Encryption Standard)
  - FIPS standard since 2002
  - 128, 192, or 256-bit keys; operates on 128-bit blocks
- DES, 3DES
  - FIPS standard since 1976
  - 56-bit key; operates on 64-bit (8-byte) blocks
  - Triple DES recommended since 1999 (112 or 168 bits)
- Blowfish
  - Key length from 23-448 bits; 64-bit blocks
- IDEA
  - 128-bit keys; operates on 64-bit blocks
  - More secure than DES but faster algorithms are available

# Communicating with symmetric cryptography

- Both parties must agree on a secret key,  $K$
- Message is encrypted, sent, decrypted at other side

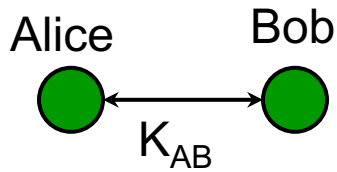


- **Key distribution must be secret**
  - otherwise messages can be decrypted
  - users can be impersonated

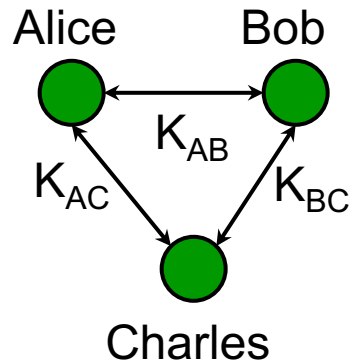
# Key Distribution

# Key explosion

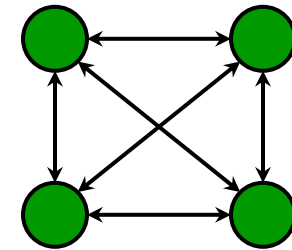
Each pair of users needs a separate key for secure communication



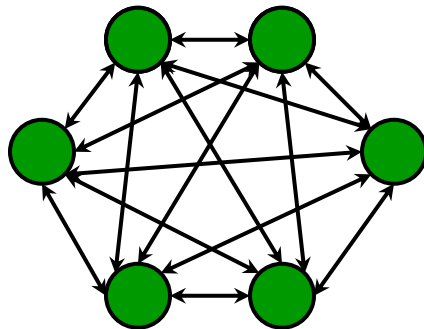
**2 users: 1 key**



**3 users: 3 keys**



**4 users: 6 keys**



**6 users: 15 keys**

100 users: 4,950 keys

1000 users: 399,500 keys

$$n \text{ users: } \frac{n(n-1)}{2} \text{ keys}$$

# Key distribution

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Secure key distribution is the biggest problem with symmetric cryptography



# Public-key algorithm

- Two related keys.

$$C = E_{K_1}(P) \quad P = D_{K_2}(C)$$

$$C' = E_{K_2}(P) \quad P = D_{K_1}(C')$$

$K_1$  is a **public** key

$K_2$  is a **private** key

- Examples:

- RSA, Elliptic curve algorithms,  
DSS (digital signature standard)

- Key length

- Unlike symmetric cryptography, not every number is a valid key
- 3072-bit RSA = 256-bit elliptic curve = 128-bit symmetric cipher
- 15360-bit RSA = 521-bit elliptic curve = 256-bit symmetric cipher

# RSA Public Key Cryptography

- Ron Rivest, Adi Shamir, Leonard Adleman created a true public key encryption algorithm in 1977
- Each user generates two keys:
  - **Private key** (kept secret)
  - **Public key** (can be shared with anyone)
- Difficulty of algorithm based on the difficulty of factoring large numbers
  - keys are functions of a pair of large (~300 digits) prime numbers

# RSA algorithm

## How to generate keys

- choose two random large prime numbers  $p, q$
- Compute the product  $n = pq$
- randomly choose the encryption key,  $e$ , such that:  
 $e$  and  $(p - 1)(q - 1)$  are relatively prime
- Compute a decryption key,  $d$  such that:  
 $ed = 1 \pmod{(p - 1)(q - 1)}$   
 $d = e^{-1} \pmod{(p - 1)(q - 1)}$
- discard  $p, q$

The security of the algorithm rests on our understanding that factoring  $n$  is extremely difficult

# RSA Encryption

- What you need:
  - Key pair:  $e, d$
  - Agreed-upon modulus:  $n$
- Encrypt:
  - divide data into numerical blocks  $< n$
  - encrypt each block:  
$$c = m^e \bmod n$$
- Decrypt:  
$$m = c^d \bmod n$$

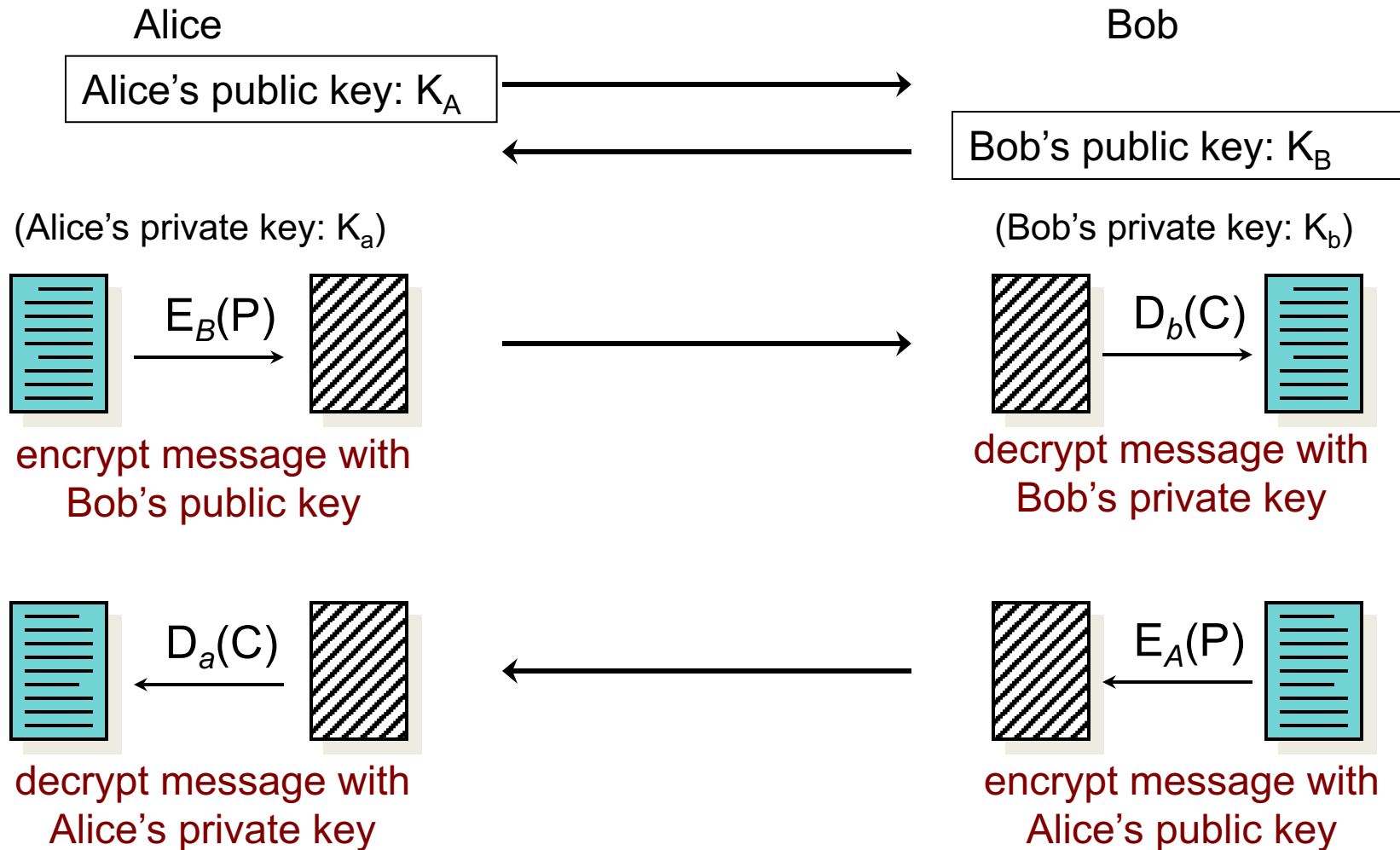
# Communication with public key algorithms

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Different keys for encrypting and decrypting

- No need to worry about key distribution

# Communication with public key algorithms



# RSA isn't good for large-scale encryption

Calculations are very expensive (& key generation is slow)

Common speeds:

Algorithm	Bytes/sec
AES-128-ECB	148,000,000
AES-128-CBC	153,000,000
AES-256-ECB	114,240,000
RSA-2048 encrypt	3,800,000
RSA-2048 decrypt	96,000

- AES ~1500x faster to decrypt; 40x faster to encrypt
- RSA is also subject to mathematical attacks
  - Certain keys (numbers) may expose weaknesses
- **If anyone learns your private key, they can read all your messages**

# Diffie-Hellman Key Exchange

## Key distribution algorithm

- Allows two parties to exchange keys securely
- Not public key encryption
- Based on difficulty of computing discrete logarithms in a finite field compared with ease of calculating exponentiation

Allows us to negotiate a secret **common key** without fear of eavesdroppers



# Diffie-Hellman Key Exchange

- All arithmetic performed in a field of integers modulo some large number
- Both parties agree on
  - a **large prime number  $p$**
  - and a **number  $\alpha < p$**
- Each party generates a public/private key pair

Private key for user  $i$ :  $X_i$

Public key for user  $i$ :  $Y_i = \alpha^{X_i} \bmod p$

# Diffie-Hellman exponential key exchange

- Alice has secret key  $X_A$
- Alice sends Bob public key  $Y_A$
- Alice computes
- Bob has secret key  $X_B$
- Bob sends Alice public key  $Y_B$

$$K = Y_B^{X_A} \bmod p$$

**$K = (\text{Bob's public key}) (\text{Alice's private key}) \bmod p$**

# Diffie-Hellman exponential key exchange

- Alice has secret key  $X_A$
- Alice sends Bob public key  $Y_A$
- Alice computes
- Bob has secret key  $X_B$
- Bob sends Alice public key  $Y_B$
- Bob computes

$$K = Y_B^{X_A} \bmod p$$

$$K = Y_A^{X_B} \bmod p$$

$$***K' = (Alice's public key) (Bob's private key) \bmod p***$$

# Diffie-Hellman exponential key exchange

- Alice has secret key  $X_A$
- Alice sends Bob public key  $Y_A$
- Alice computes

$$K = Y_B^{X_A} \bmod p$$

- expanding:

$$\begin{aligned} K &= Y_B^{X_A} \bmod p \\ &= (\alpha^{X_B} \bmod p)^{X_A} \bmod p \\ &= \alpha^{X_B X_A} \bmod p \end{aligned}$$

- Bob has secret key  $X_B$
- Bob sends Alice public key  $Y_B$
- Bob computes

$$K = Y_A^{X_B} \bmod p$$

- expanding:

$$\begin{aligned} K &= Y_A^{X_B} \bmod p \\ &= (\alpha^{X_A} \bmod p)^{X_B} \bmod p \\ &= \alpha^{X_A X_B} \bmod p \end{aligned}$$

$$\mathbf{K = K'}$$

$K$  is a common key, known *only* to Bob and Alice

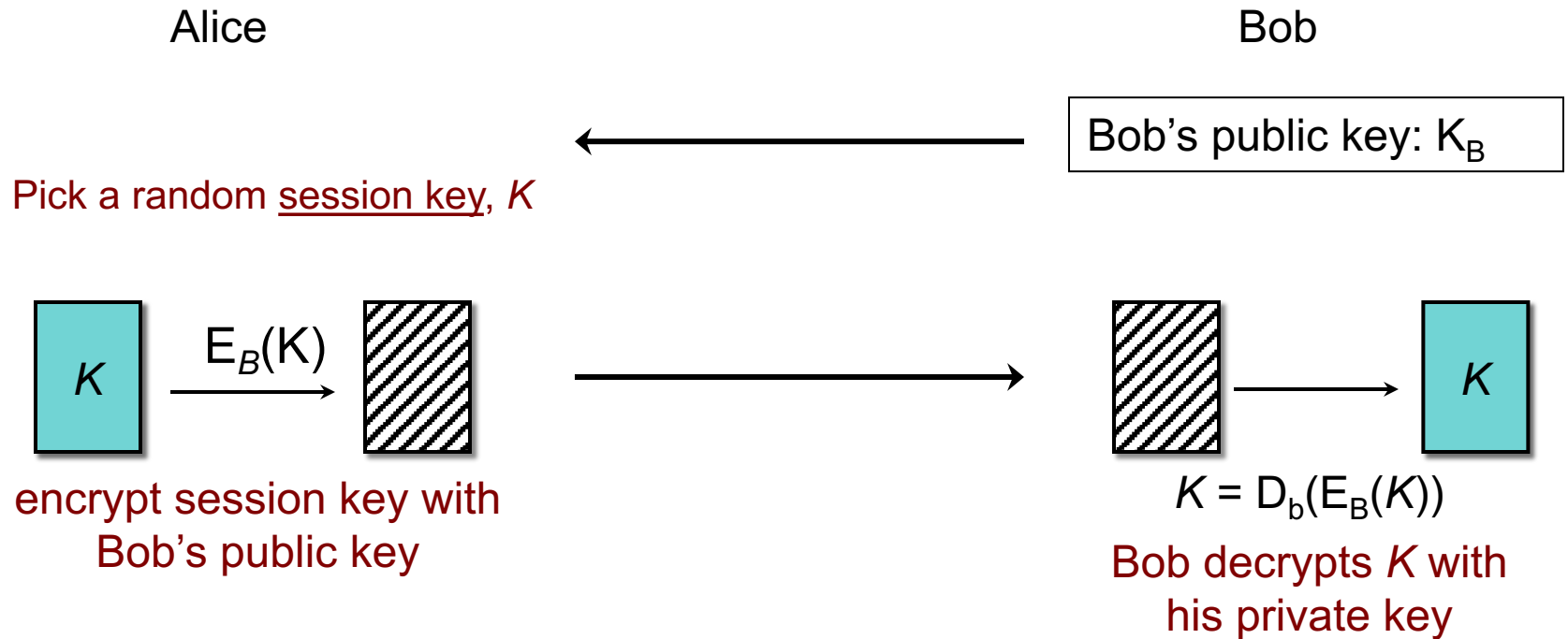
# Hybrid Cryptosystems

- **Session key**: randomly-generated key for one communication session
- Use a **public key algorithm** to send the session key
- Use a **symmetric algorithm** to encrypt data with the session key

Public key algorithms are almost never used to encrypt messages

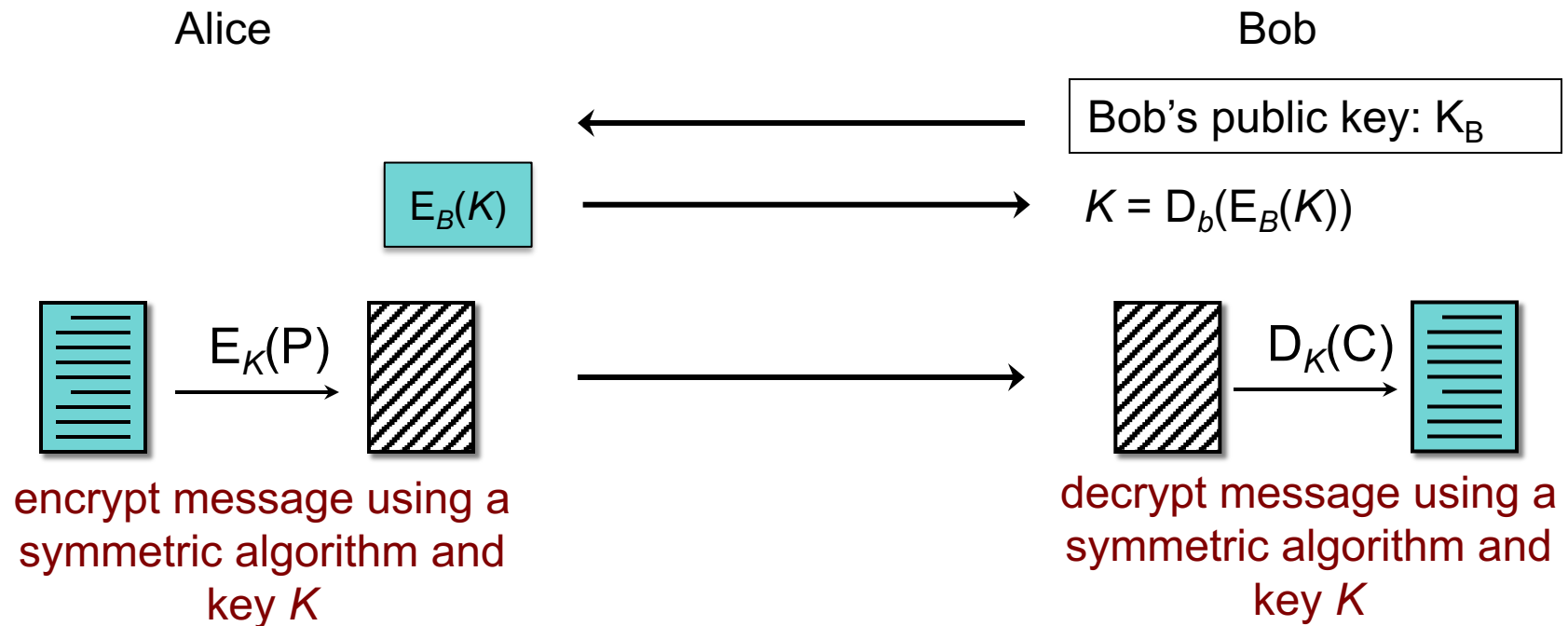
- Vulnerable to chosen plaintext attacks
- MUCH slower; RSA-2048 approximately 40x slower to encrypt and 1,500x slower to decrypt than AES-256

# Communication with a hybrid cryptosystem

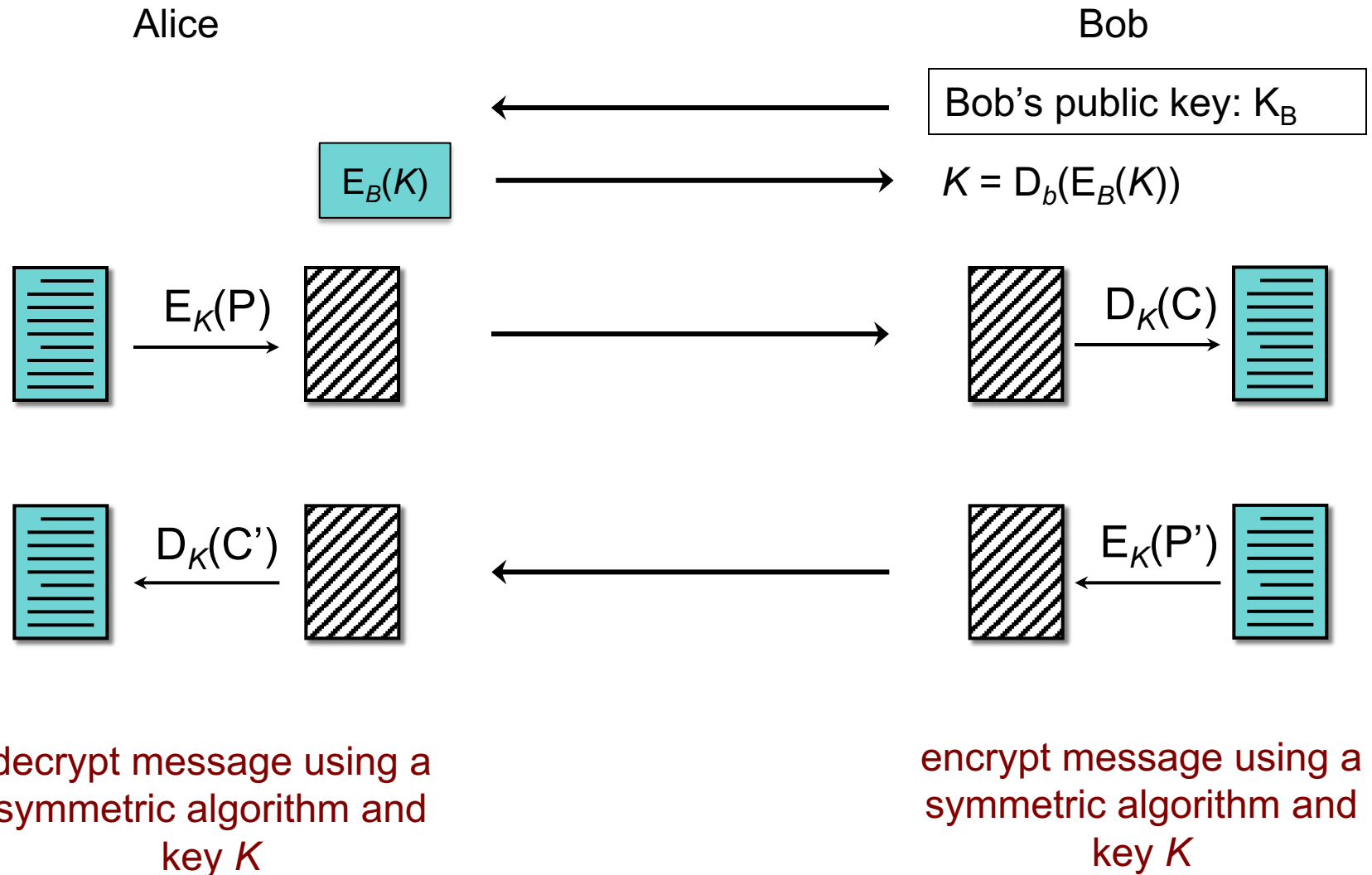


Now Bob knows the secret session key,  $K$

# Communication with a hybrid cryptosystem



# Communication with a hybrid cryptosystem





# Forward Secrecy

Suppose an attacker steals Bob's private key

- Future messages can be compromised
- But past messages that used the key can also be decrypted

Pick a session key  
Encrypt it with the Bob's public key

Bob decrypts the session key

Security rests entirely on the secrecy of Bob's private key

If Bob's private key is compromised, all recorded past traffic can be decrypted

# Forward Secrecy

- **Forward secrecy**
  - *Compromise of long term keys does not compromise past session keys*
  - There is no one secret to steal that will compromise multiple messages
- **Diffie-Hellman**
  - Use common key as the encryption/decryption key
    - Or as a key to encrypt a session key
  - Not recoverable as long as long as private keys are thrown away after each session
  - Unlike RSA keys, Diffie Hellman makes key generation simple
- **Keys must be ephemeral**
  - Client & server will generate new Diffie-Hellman parameters for each session – all will be thrown away after the session

Diffie-Hellman is preferred over RSA for key exchange to achieve forward secrecy – generating Diffie-Hellman keys is a rapid, low-overhead process

# Cryptographic systems: summary

- **Symmetric ciphers**
  - Based on “SP networks” = substitution & permutation sequences
- **Asymmetric ciphers** – public key cryptosystems
  - Based on **trapdoor** functions  
Easy to compute in one direction; difficult to compute in the other direction without special information (the trapdoor)
- **Hybrid cryptosystem**
  - Pick a random session key
  - Use a public key algorithm to send
  - Use a symmetric key algorithm to encrypt traffic back & forth
- Key exchange algorithms (more to come later)
  - Diffie Hellman
  - Public key

*Enables secure communication without knowledge of a shared secret*

# RSA cryptography in the future

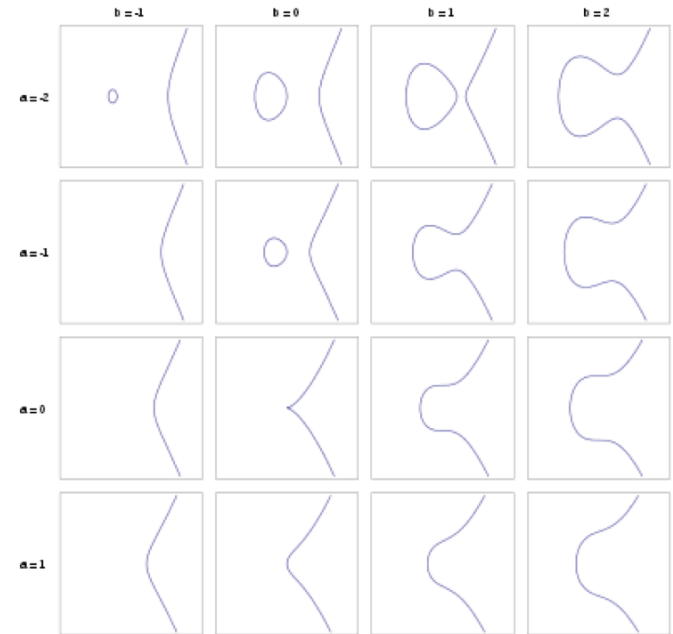
- Based on the difficulty of factoring products of two large primes
- Factoring algorithms get more efficient as numbers get larger
  - As the ability to decrypt numbers increases, the key size must therefore grow even faster
  - This is not sustainable (especially for embedded devices)

# Elliptic Curve Cryptography

- Alternate approach: elliptic curves

$$y^2 = x^3 + ax + b$$

- Using discrete numbers, pick
  - A prime number as a maximum (modulus)
  - A curve equation
  - A public base point on the curve
  - A random private key
  - Public key is derived from the private key, the base point, and the curve
- To compute the private key from the public,
  - We need an elliptic curve discrete logarithm function
  - This is difficult and is the basis for ECC's security



Catalog of elliptic curves  
[https://en.wikipedia.org/wiki/Elliptic\\_curve](https://en.wikipedia.org/wiki/Elliptic_curve)

# ECC vs. RSA

- **RSA is still the most widely used public key cryptosystem**
  - Mostly due to inertia & widespread implementations
  - Simpler implementation & faster decryption
- **ECC offers higher security with fewer bits than RSA**
  - ECC is also faster (for key generation & encryption) and uses less memory
  - NIST defines 15 standard curves for ECC
    - Many implementations support only a couple (P-256, P-384)
- **For long-term security**

The European Union Agency for Network and Information Security (ENISA) and the National Institute for Science & Technology (NIST) recommend:

  - AES: 256 bit keys
  - RSA: 15,360 bit keys
  - ECC: 512 bit keys

<https://www.keylength.com/en/4/>

<http://https://www.enisa.europa.eu/publications/algorithms-key-size-and-parameters-report-2014>

# Quantum Computers

- Once (if) real quantum computers can be built, they can
  - Crack a symmetric cipher in time proportional to the square root of the key space size:  $2^{n/2}$ 
    - Use 256-bit AES to be safe
  - Factor efficiently
    - RSA and many elliptic curve algorithms will not be secure anymore
- NSA called for a migration to “post-quantum cryptographic algorithms” – but no agreement yet on what they are

The End